

# The Oscillation Period of Gaseous Spheres

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Any collection of matter that is governed by the force gravity has a natural period of oscillation. For example, a simple pendulum such as a playground swing, will move back and forth with a time period in seconds,  $T$ , defined by the formula to the right, where  $L$  is the length of the swing in centimeters, and  $g$  is the acceleration of gravity at Earth's surface given by  $980 \text{ cm/sec}^2$ . The period of a swing that is 3 meters long is then  $T = 3.5$  seconds.

$$T = 2\pi \sqrt{\frac{L}{g}}$$

This behavior also applies to any body held together by gravity whether it is a star or a planet. The natural oscillation period of such bodies is given by the formula to the right, where  $D$  is the density of the body in  $\text{grams/cm}^3$  and  $G$  is the Newtonian constant of Gravity  $G = 6.6 \times 10^{-8}$  and  $T$  is in seconds. For example, the planet Jupiter has a density of about  $1.3 \text{ gm/cm}^3$  so its period will be about 10,500 seconds or  $T = 3$  hours. From the information below, calculate the natural periods for the various astronomical bodies.

$$T^2 = \frac{3\pi}{G D}$$



Courtesy SOHO-EIT

The sun is about 1.5 million kilometers across, and has an average density near its surface of about  $10^{-7} \text{ grams/cm}^3$

$T =$  \_\_\_\_\_ hours.



Courtesy NASA-Apollo

The Earth has a diameter of about 12,500 kilometers, and has an average density of about  $5.5 \text{ grams/cm}^3$

$T =$  \_\_\_\_\_ minutes.



Courtesy NASA-Dana Berry

A neutron star is about 50 kilometers in diameter, and has an average density of about  $2 \times 10^{14} \text{ grams/cm}^3$

$T =$  \_\_\_\_\_ seconds.

## Inquiry Question.

Since 1964, astronomers have studied objects called pulsars. These objects change their brightness from about once a second to 30 times a second in a periodic manner.

What would be a candidate object for a pulsar if the pulsar changes were due to pulsation?

What would be a candidate object of the brightness variations were due to the rotation of the body, and how does this relate to break-up speed?

Star break-up speed :  
500 km/sec

Planet break-up speed :  
10 km/sec

### Answer Key:

The sun is about 1.5 million kilometers across, and has an average density near its surface of about  $10^{-7}$  grams/cm<sup>3</sup>

$$\begin{aligned} T &= 3.7 \times 10^7 \text{ seconds} \\ &= 10,500 \text{ hours.} \end{aligned}$$

The Earth has a diameter of about 12,500 kilometers, and has an average density of about 5.5 grams/cm<sup>3</sup>

$$T = 85 \text{ minutes.}$$

A neutron star is about 50 kilometers in diameter, and has an average density of about  $2 \times 10^{14}$  grams/cm<sup>3</sup>

$$T = 0.001 \text{ seconds.}$$

#### Inquiry Question.

Students will probably *GOOGLE* 'pulsars' and learn that they are neutron stars. They should, however, notice that for pulsation, only neutron stars have a period (0.001 seconds) that is close enough to the 1 second to 0.033 second brightness changes cited for pulsars. Planets and stars are just too low-density to make pulsations that fast.

Rotating bodies cannot spin faster than their break up speed. For a star like the sun, this is about 500 km/sec, and for planets like earth is about 10 Km/sec. These speeds are too low to allow normal stars and planets to cause the fast pulsar changes.